The Chaining Lemma and its Application

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joint work with

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The starting point: A basic question

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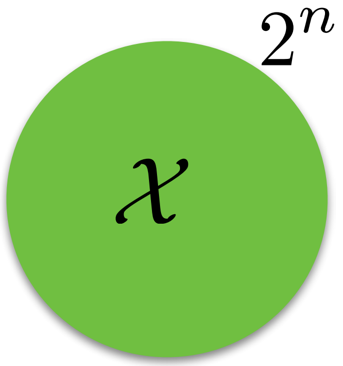

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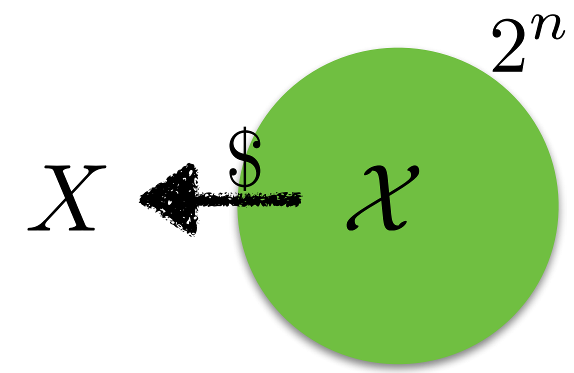

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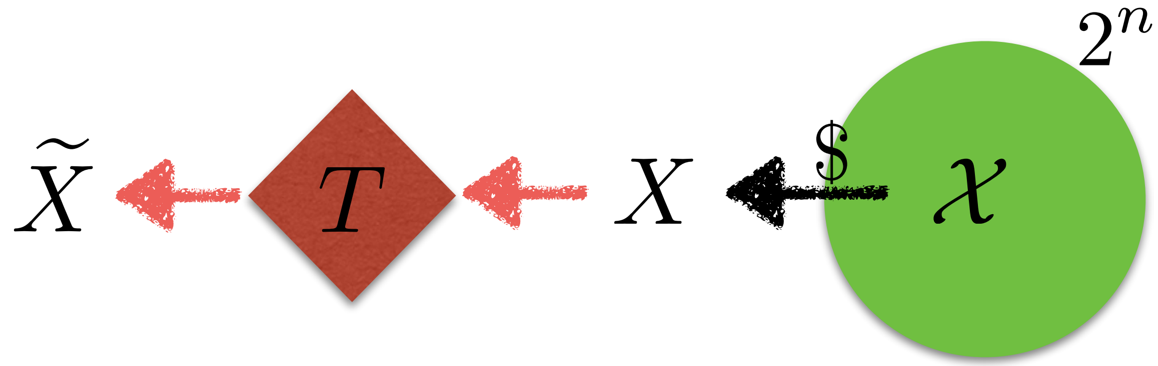

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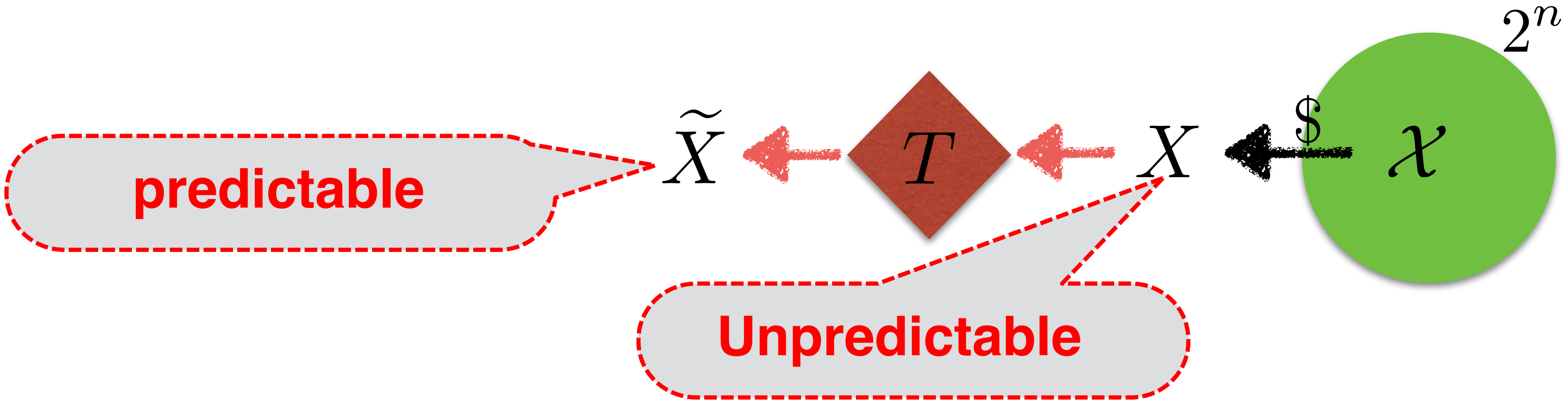

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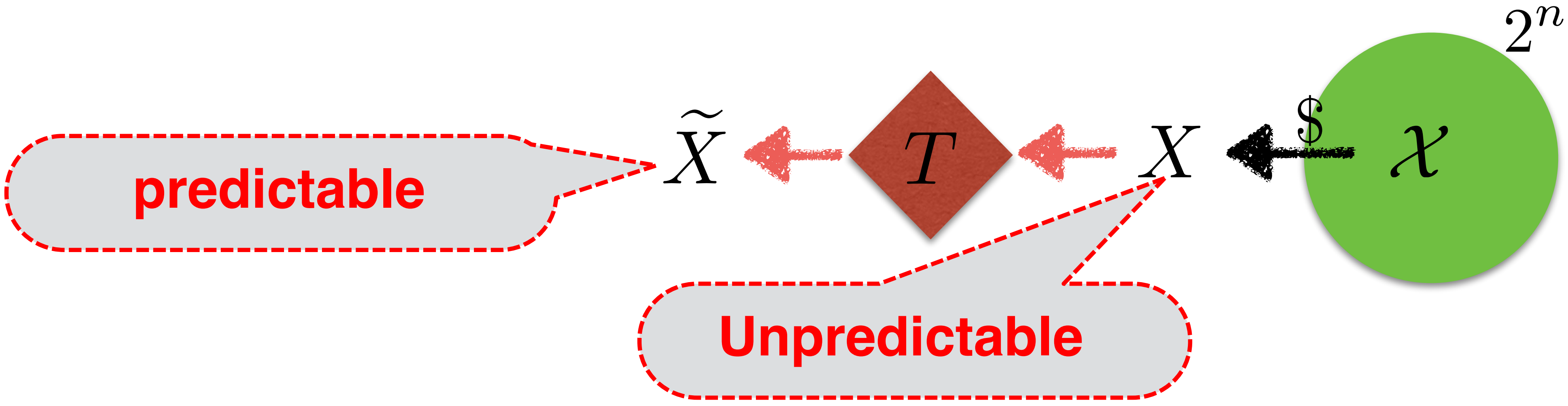

Natural Question:

How much $\widetilde{X}$ reveals about X ?

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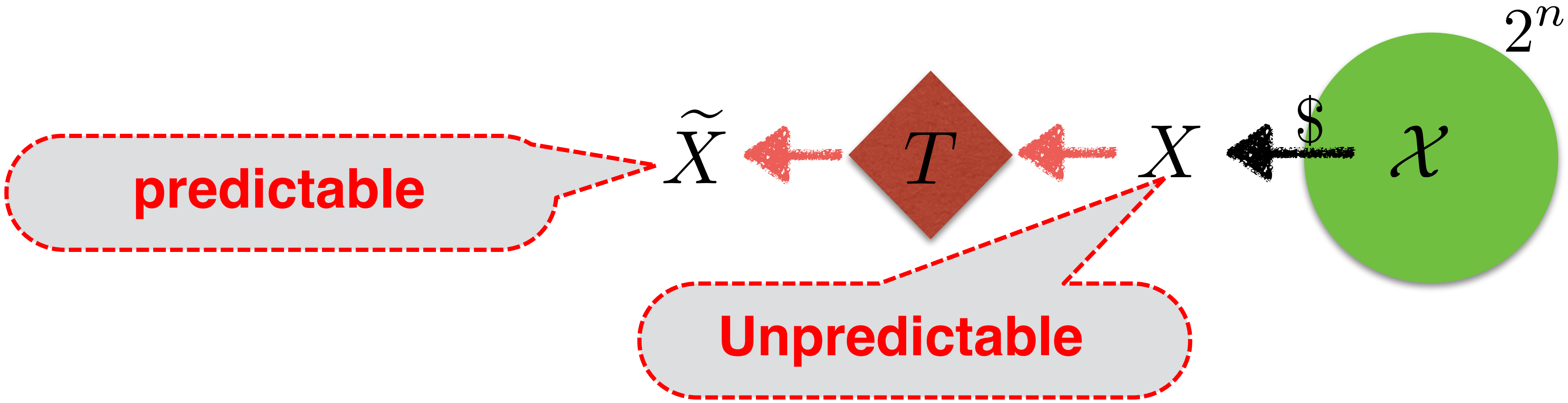

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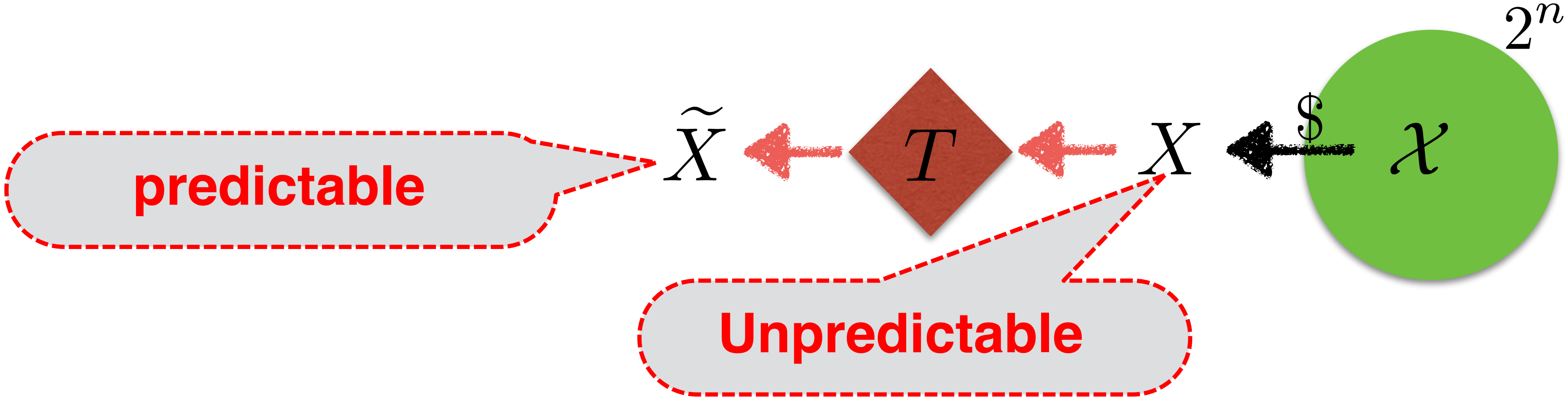

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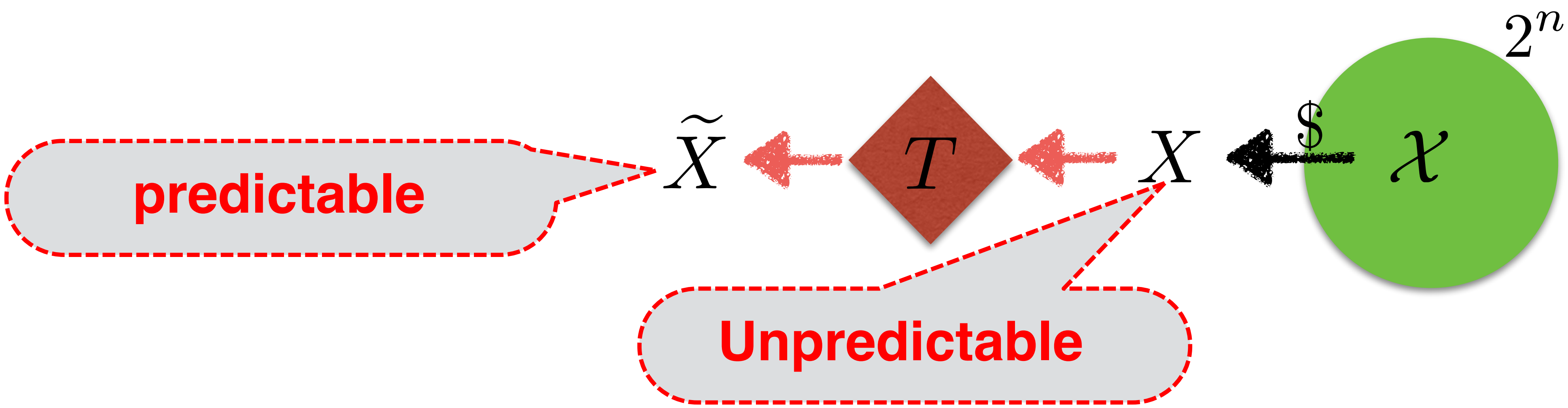

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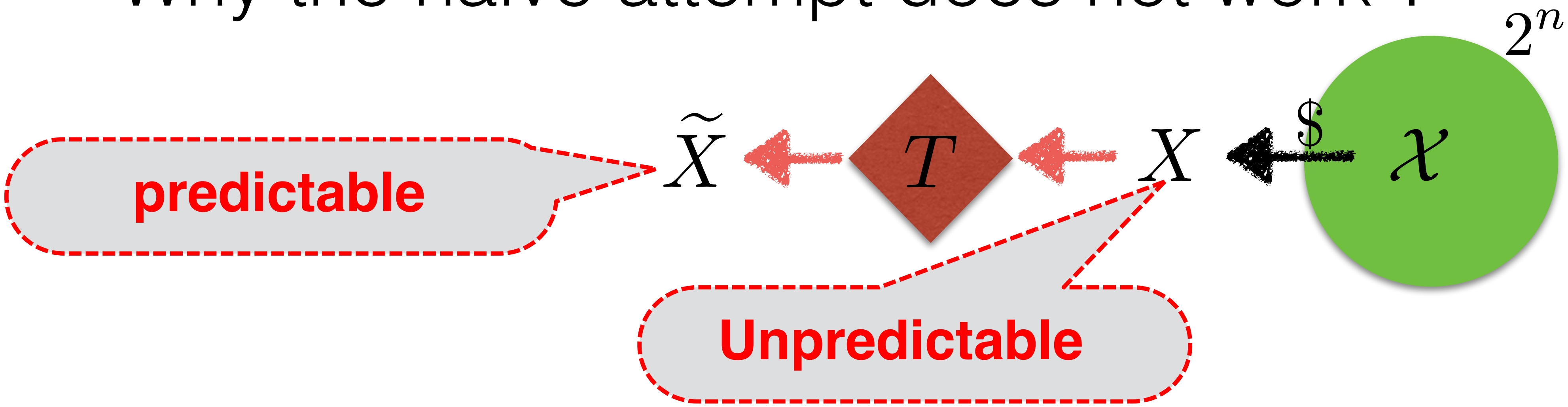
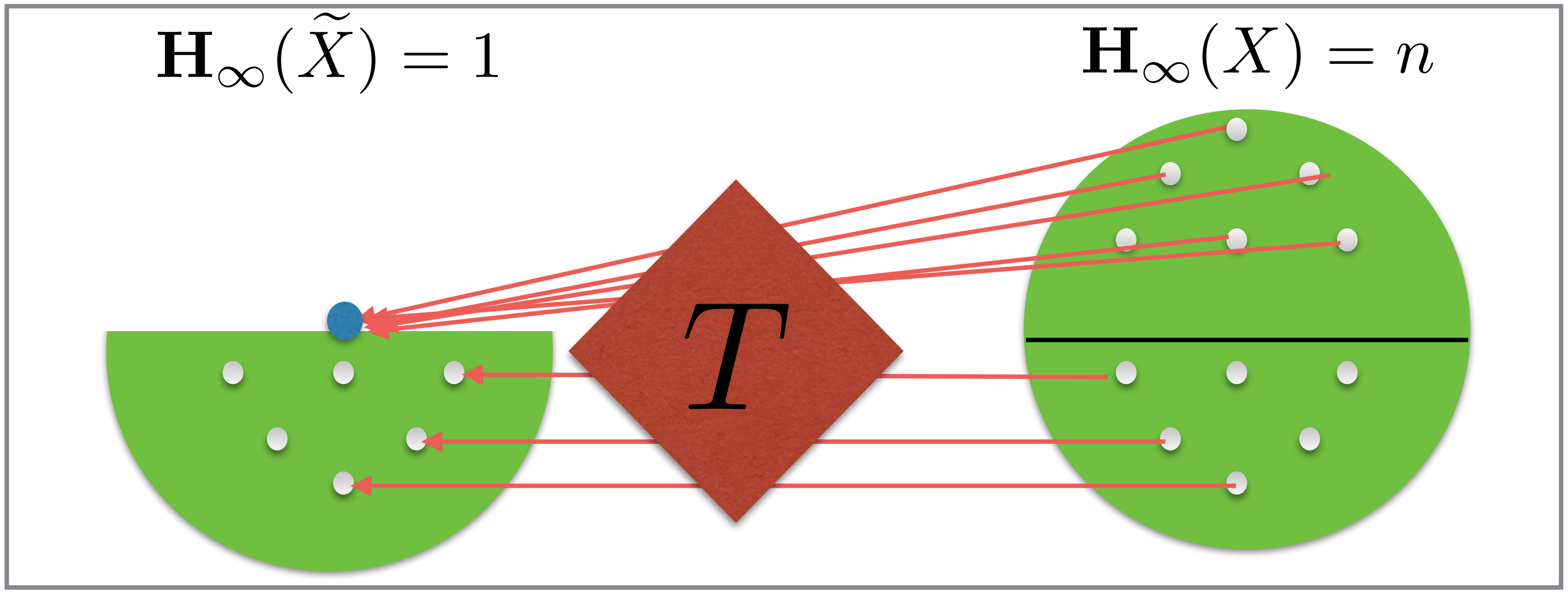
$\mathbf{H}_\infty(X) := -\log \max_x \Pr[X = x]$

An example T :
Why the naive attempt does not work ?





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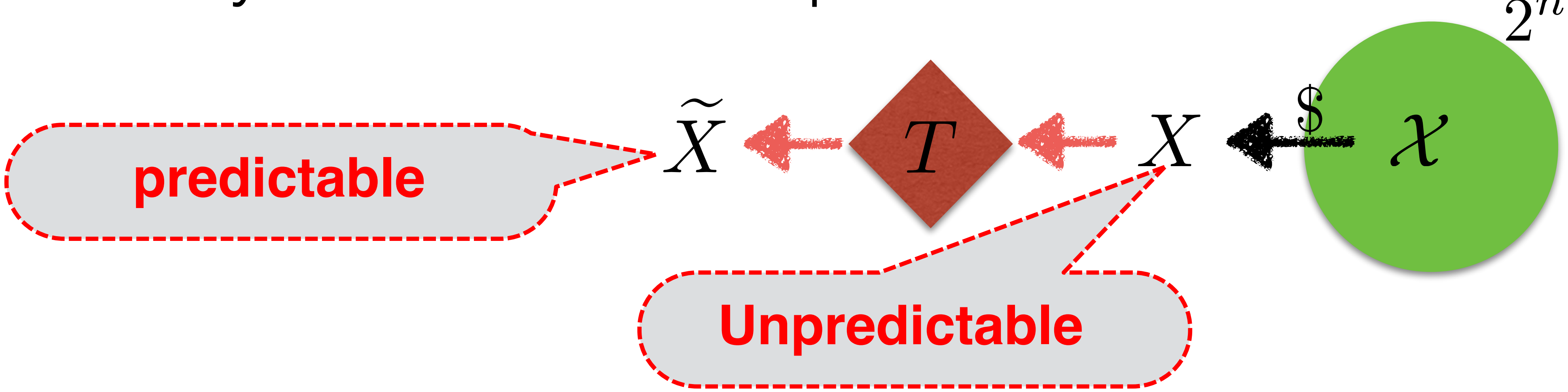
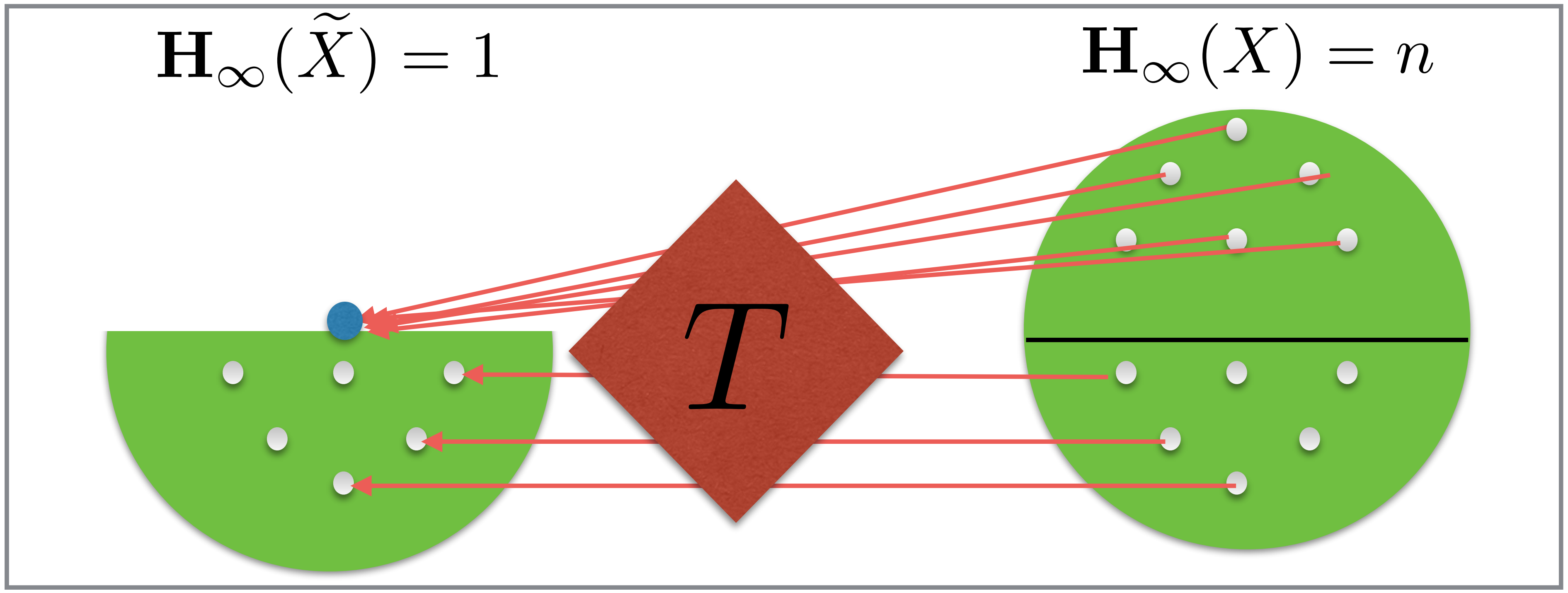
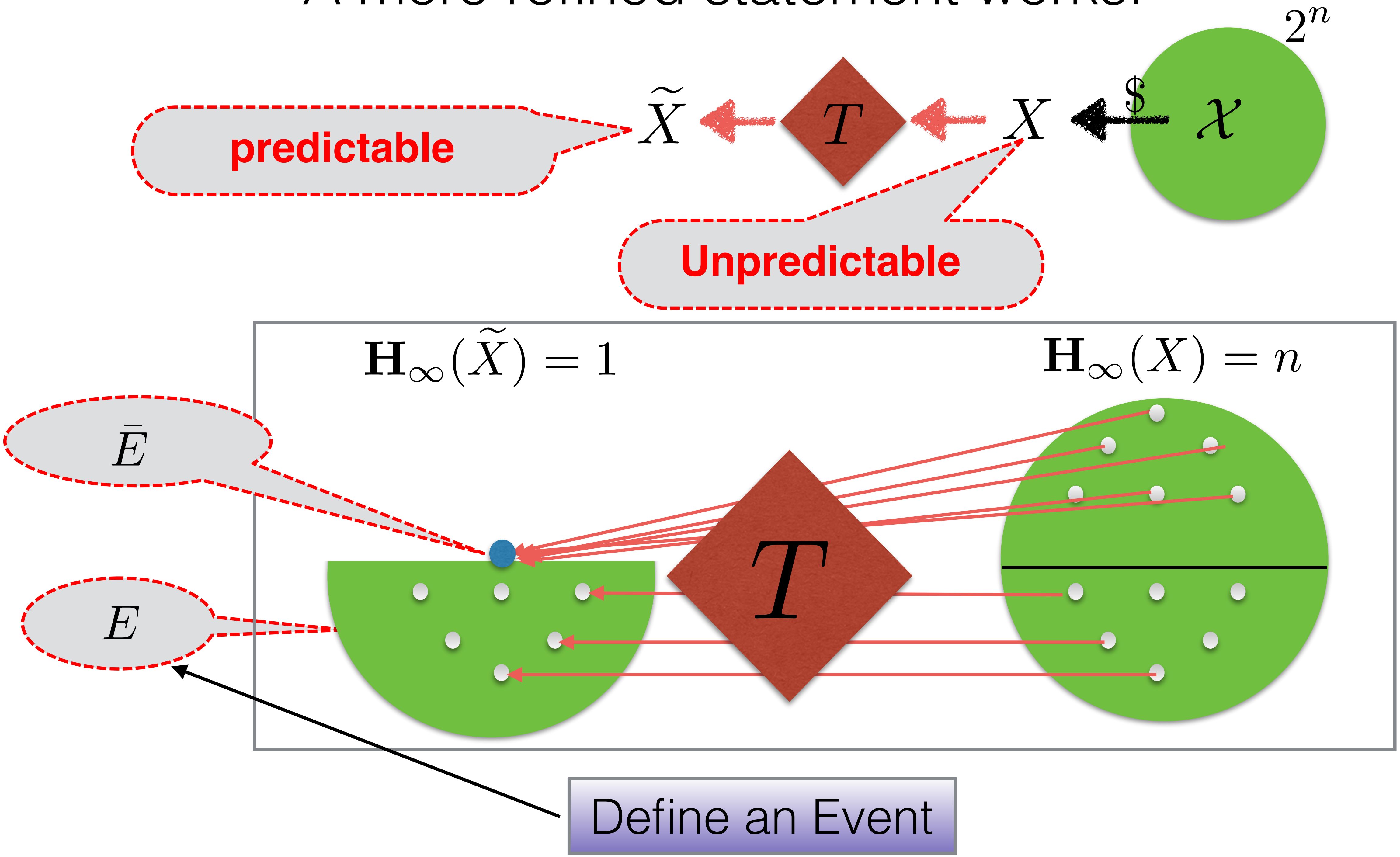




Half the times reveals everything

An example T :
A more refined statement works.



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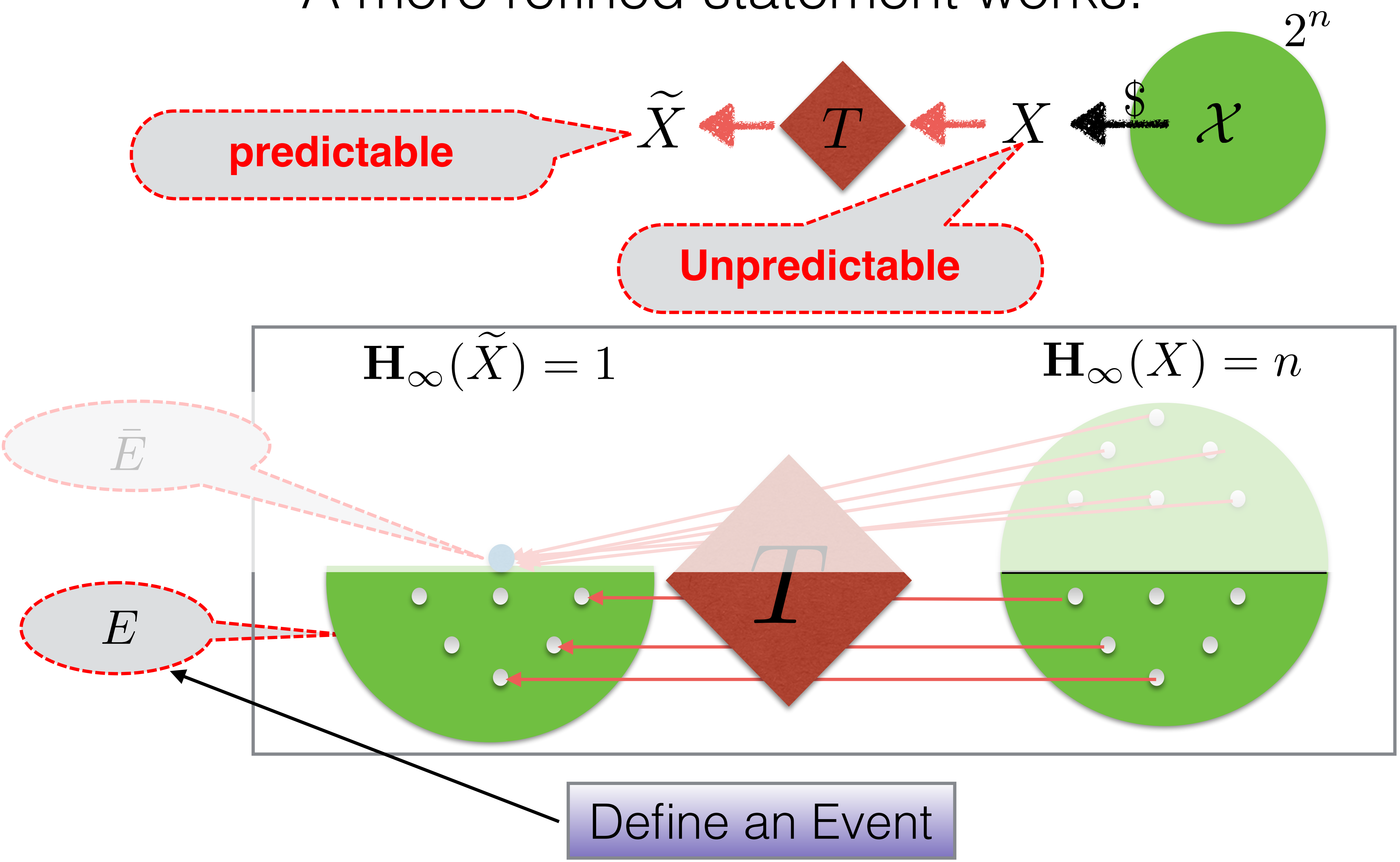

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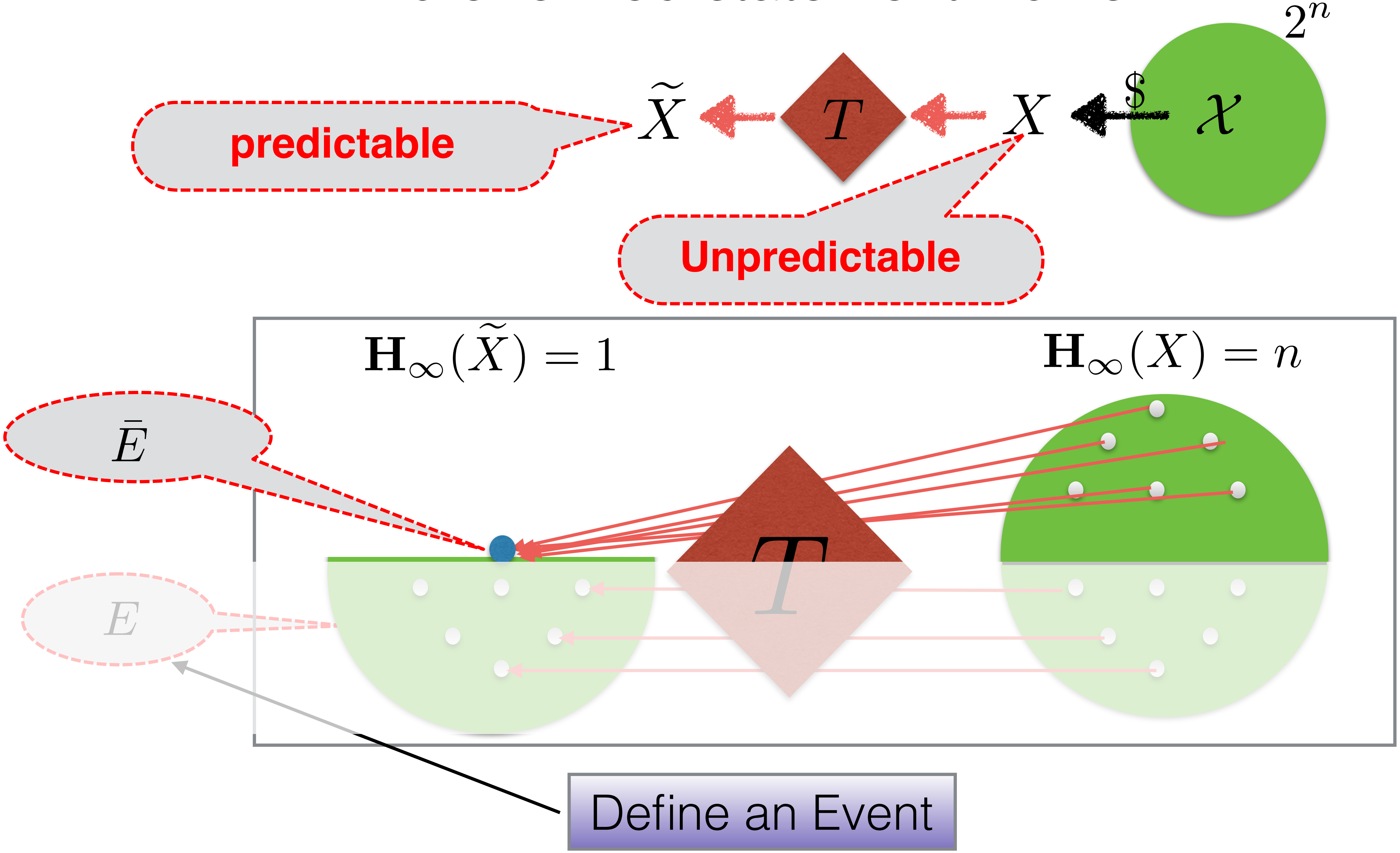
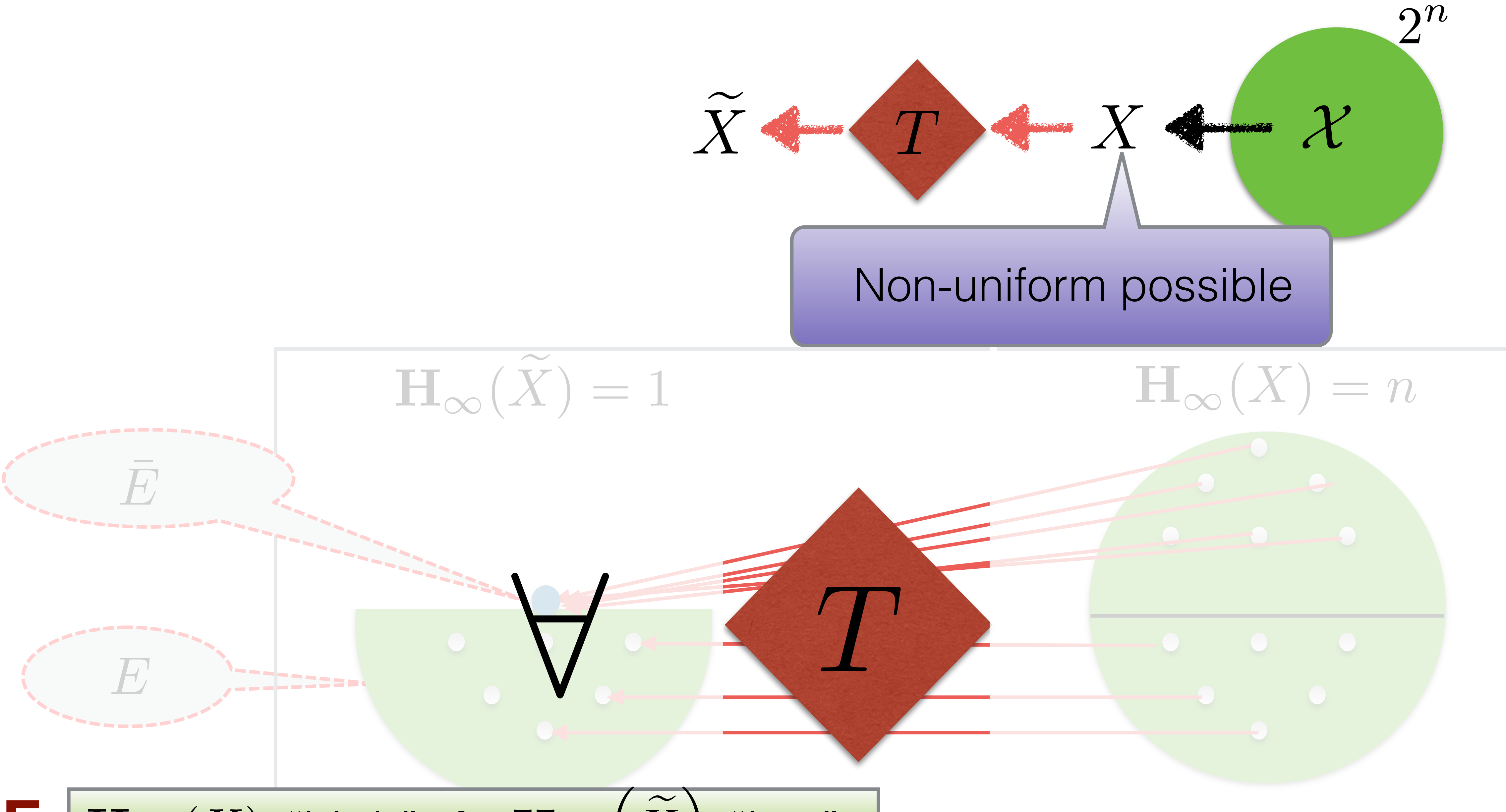


2. When $\bar{E}$ happens then $X|_{\bar{E}} \mid \widetilde{X}|_{\bar{E}}$ has “high” min-entropy.

The basic conjecture



IF $\mathbf{H}_\infty(X)$ "high" & $\mathbf{H}_\infty\left(\widetilde{X}\right)$ "low" **THEN** There exists Event E

1. When E happens then both $\widetilde{X}|_E$ and $X|_E$ has "high" min-entropy
2. When $\bar{E}$ happens then $X|_{\bar{E}}$ | $\widetilde{X}|_{\bar{E}}$ has "high" min-entropy.

Generalization over the Chain

Uniform

The Chain

$$X^{(0)} \xrightarrow{T_1} X^{(1)} \xrightarrow{T_2} X^{(2)} \ldots \xrightarrow{T_t} X^{(t)}$$

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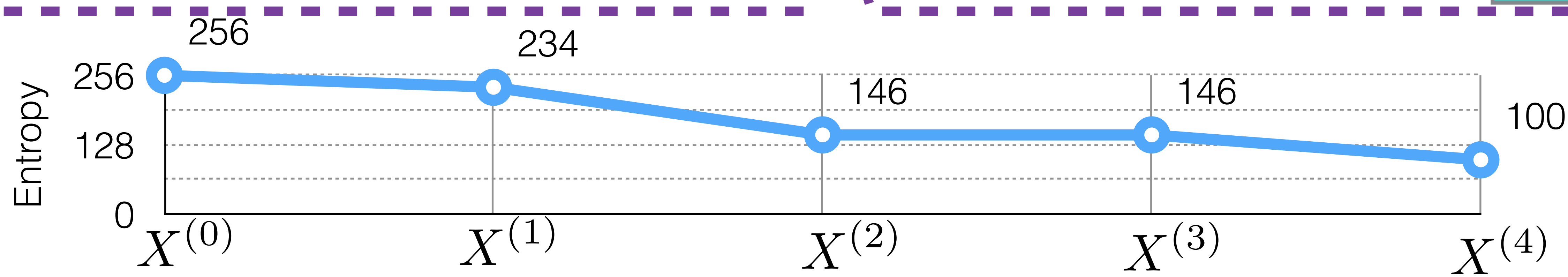
Fact

$$\mathbf{H}_\infty(X^{(i)}) \le \mathbf{H}_\infty(X^{(i-1)})$$

e.g.
$t = 4$
$n = 256$



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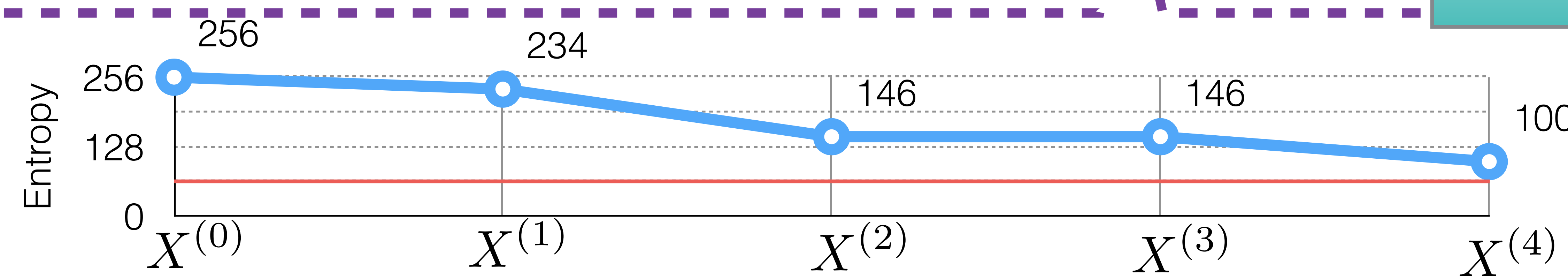

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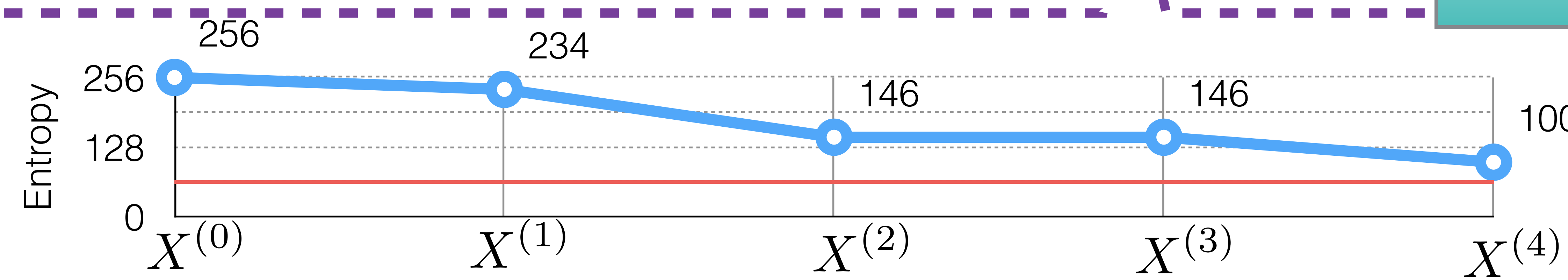

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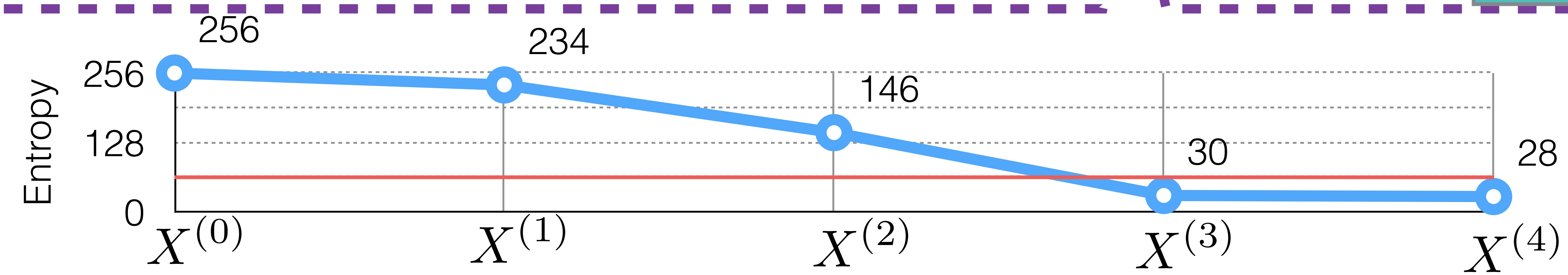

Question:

Can we "save" part of the chain ?

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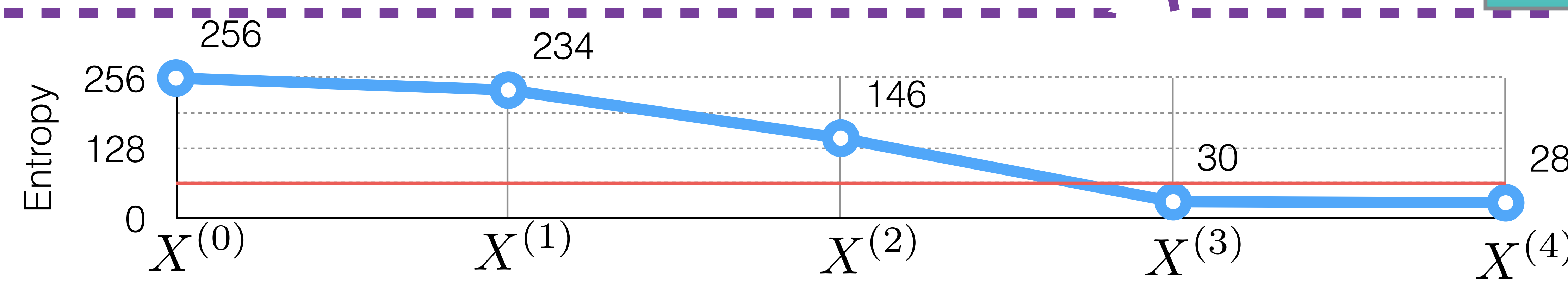

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$i \neq j$ possible

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$t = O(\sqrt{n})$

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$$\mathbf{H}_\infty\left(X^{(t)}_{|E}\right) \geq u$$

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$$\widetilde{\mathbf{H}}_\infty\left(X^{(i-1)}_{|\bar{E}} \mid X^{(i)}_{|\bar{E}}\right) \geq u$$

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$$\widetilde{\mathbf{H}}_\infty(X|Z) := -\log \mathbb{E}_{z \leftarrow Z}[2^{-\mathbf{H}_\infty(X|Z=z)}]$$

DORS '08

Some intuitions

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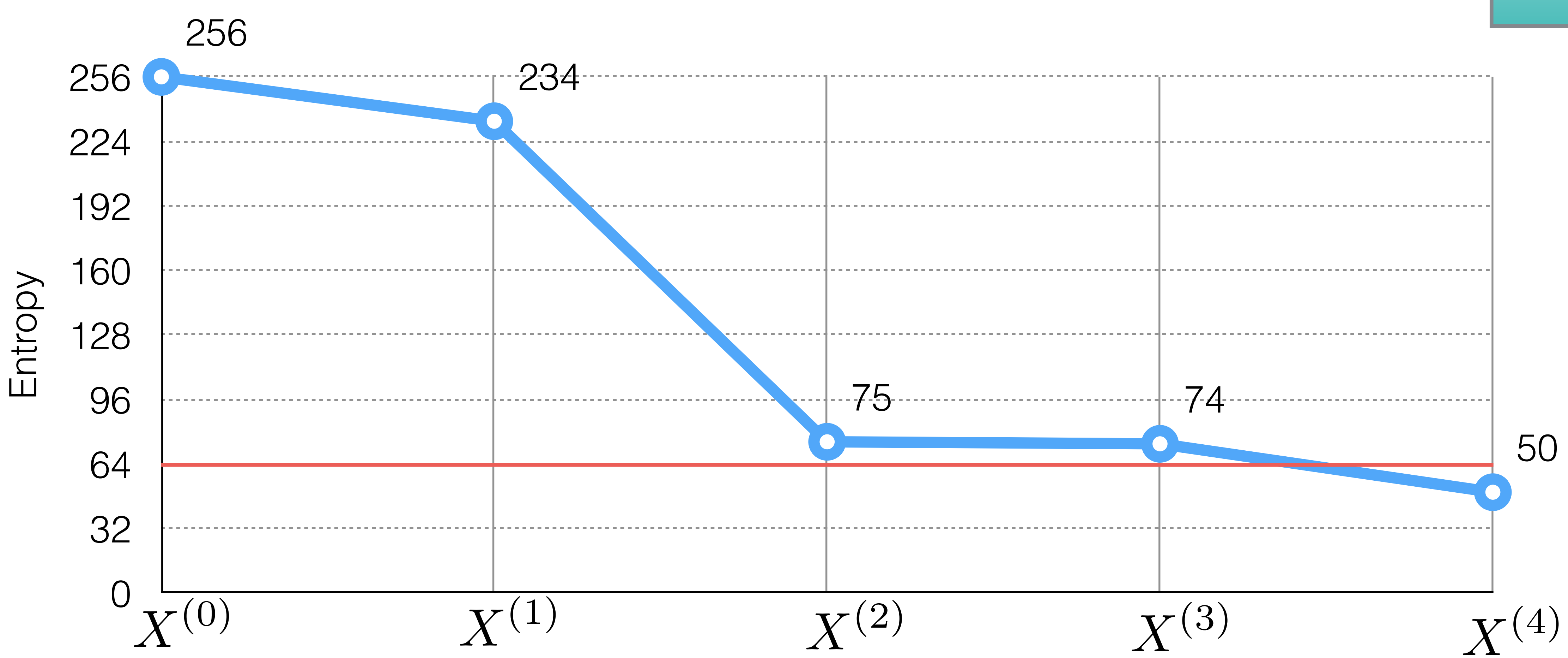

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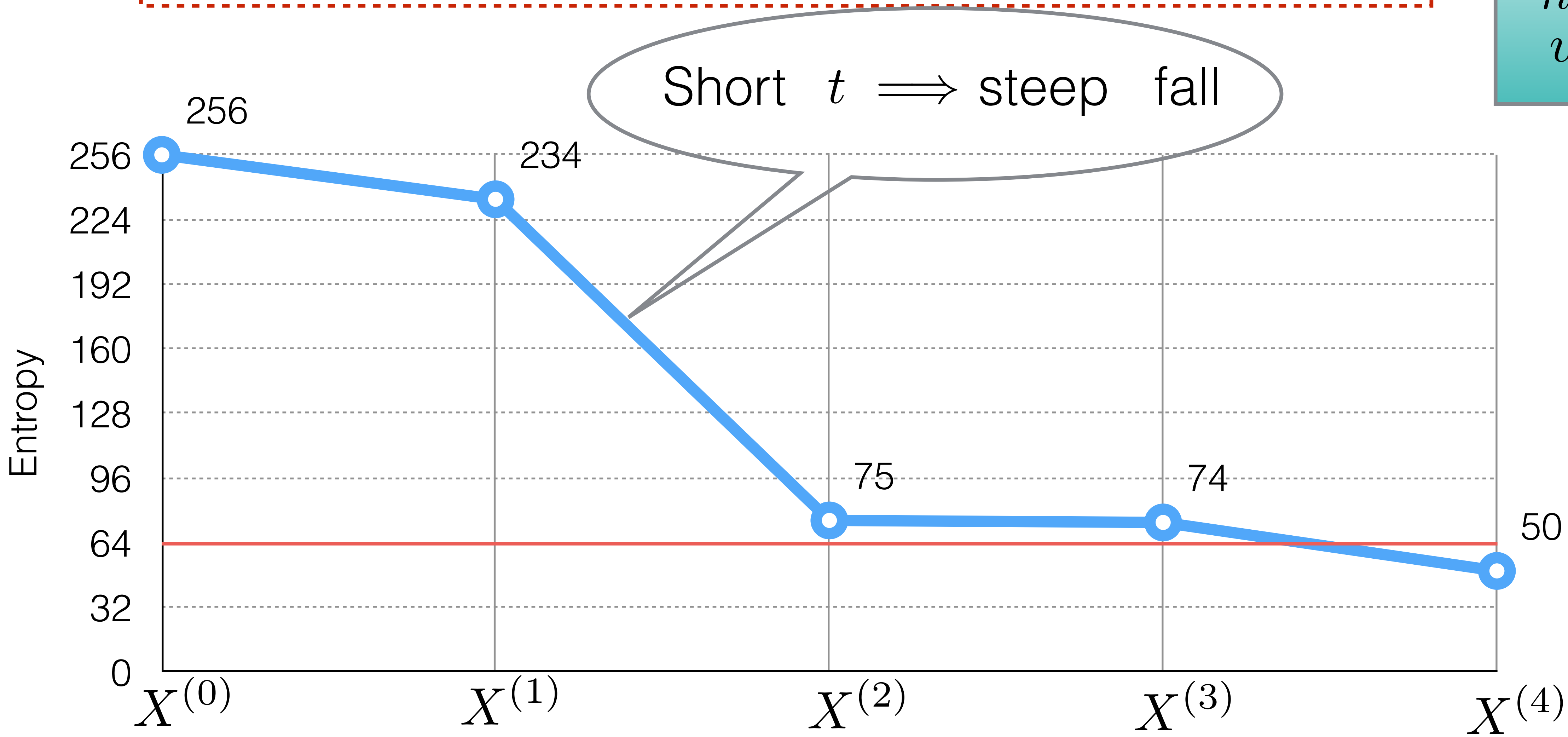

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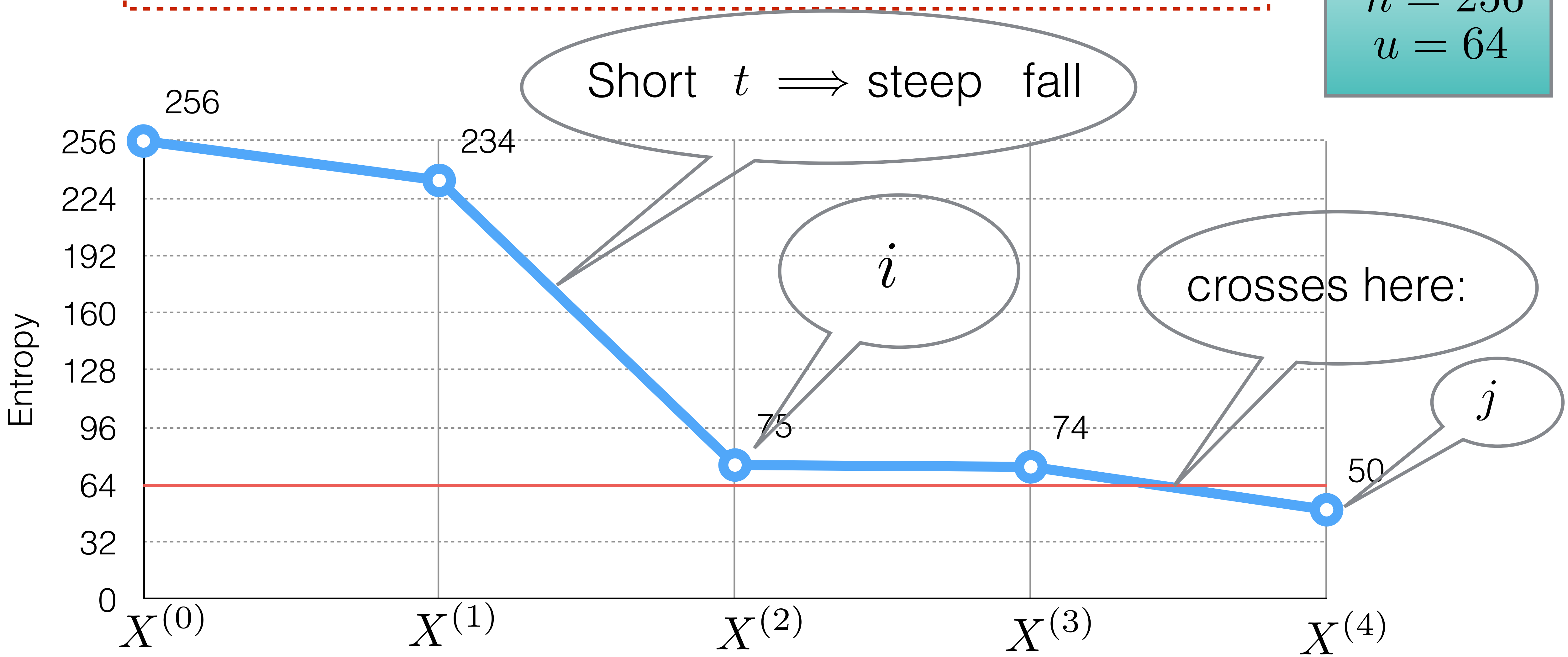

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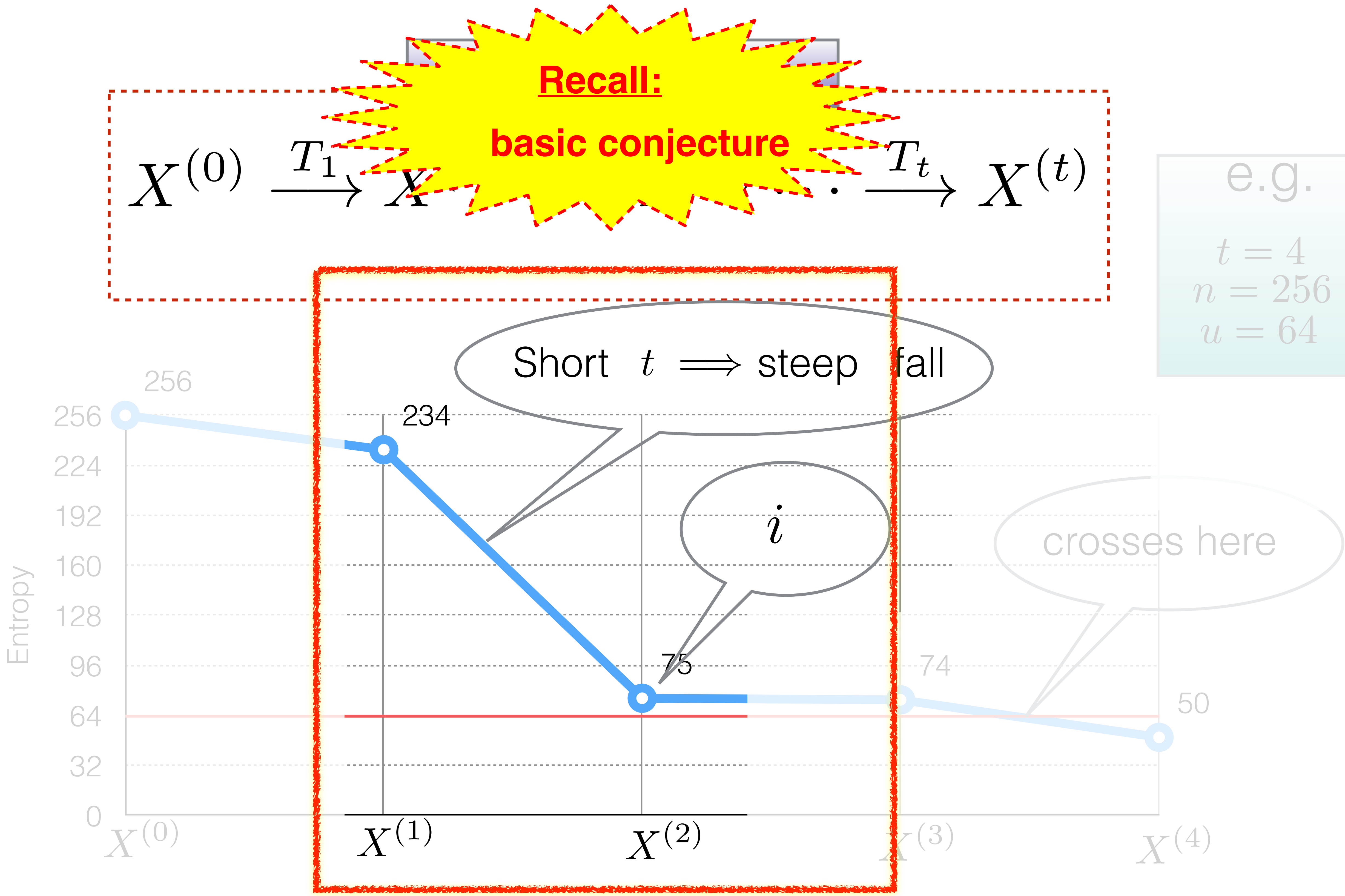
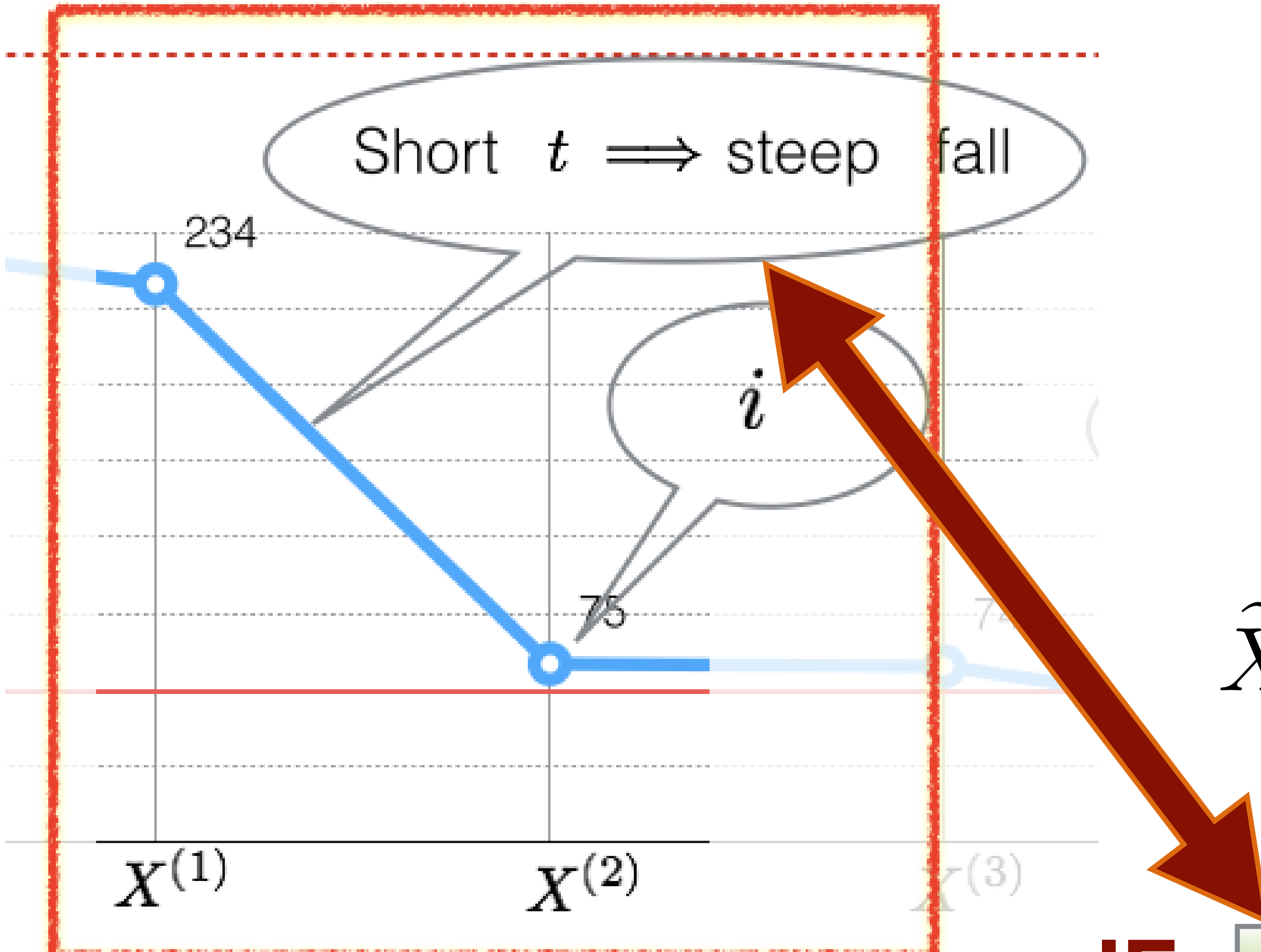
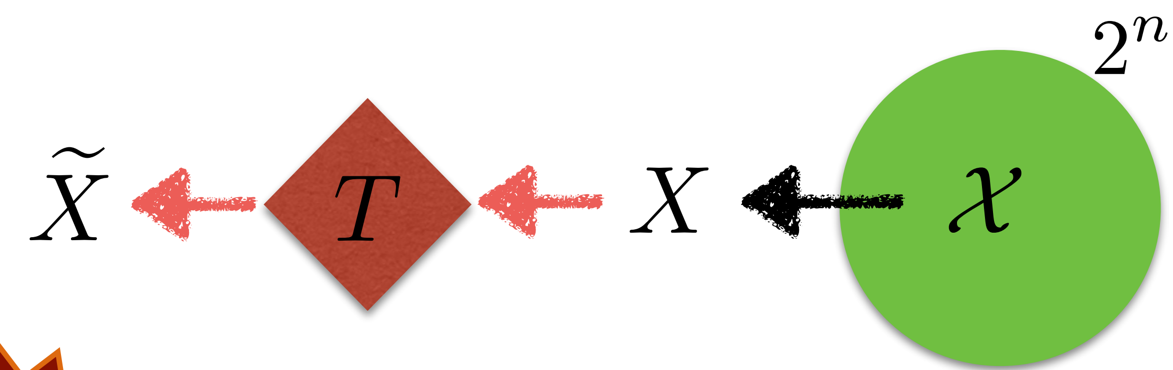


Recall the basic conjecture





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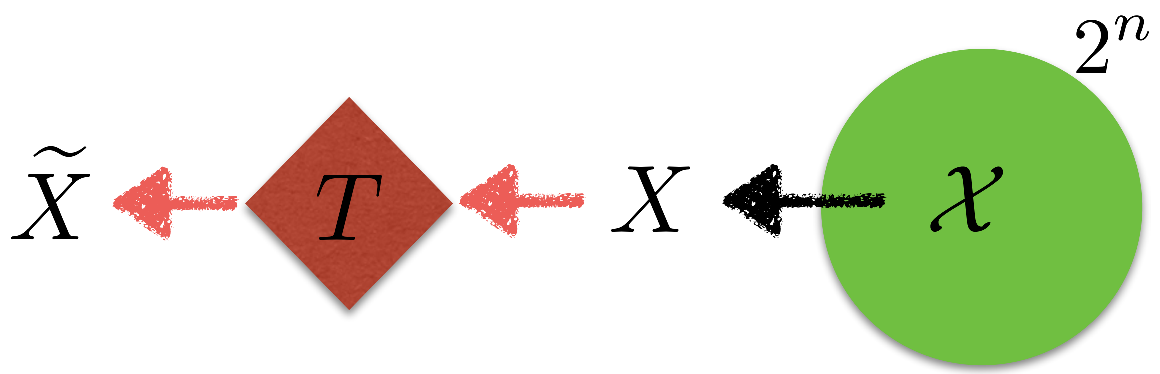
Proof overview of the basic conjecture

Key-question

how to define such event for any general function ?



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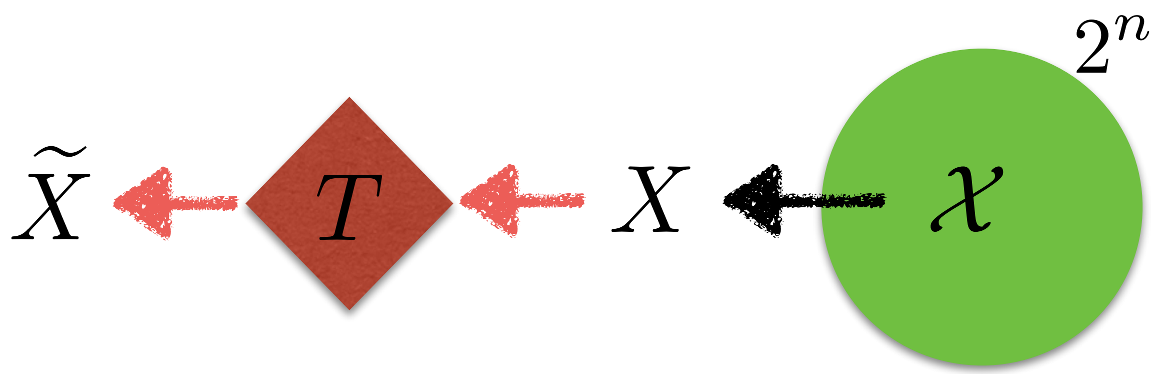

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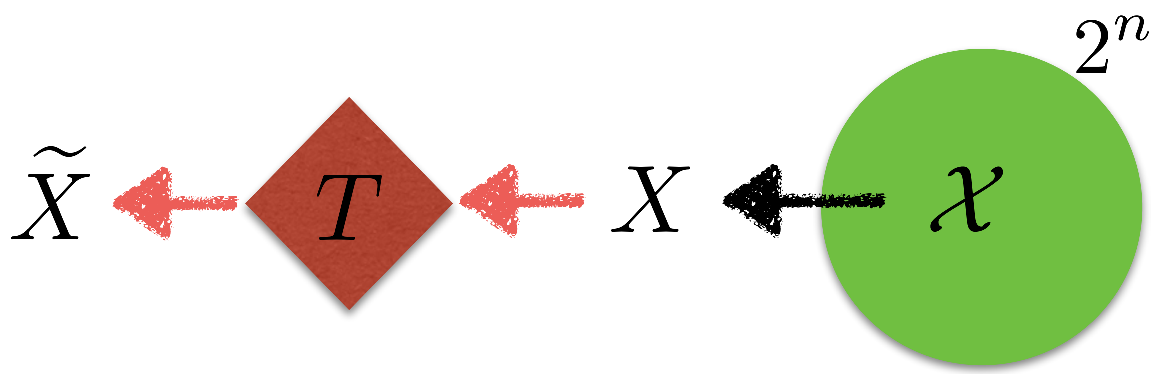

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[DORS '08]

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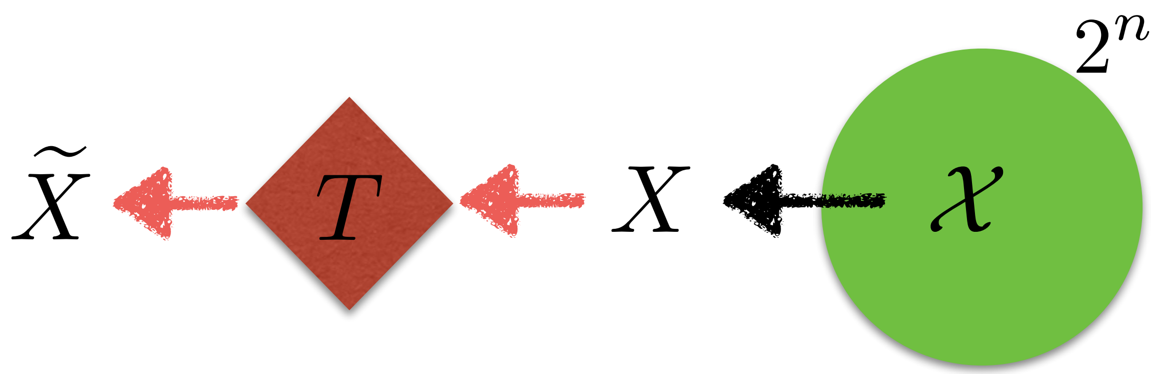

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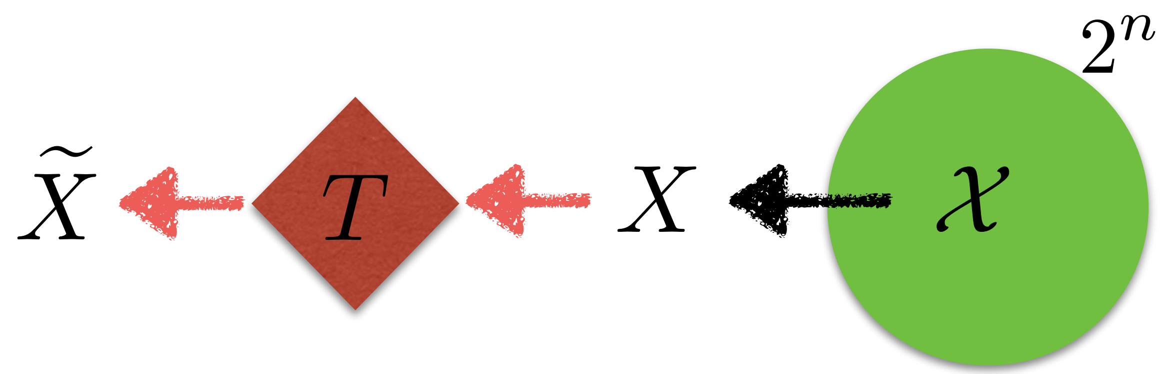

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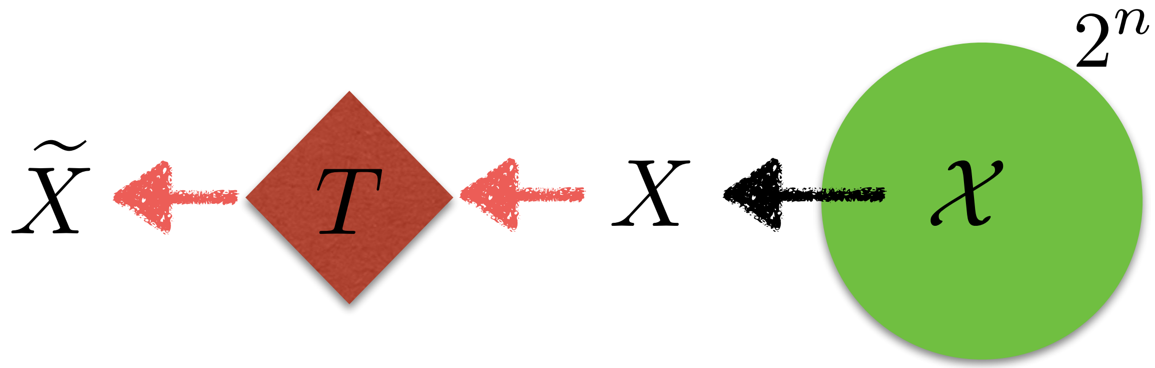

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Lemma (Flat Area)

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Proof Intuitions:

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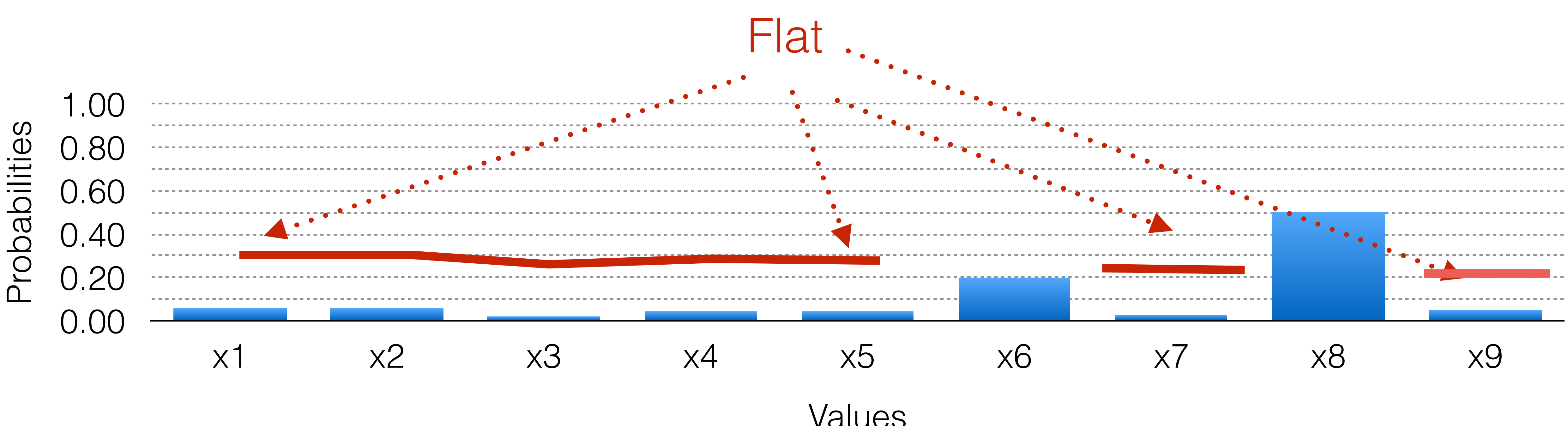

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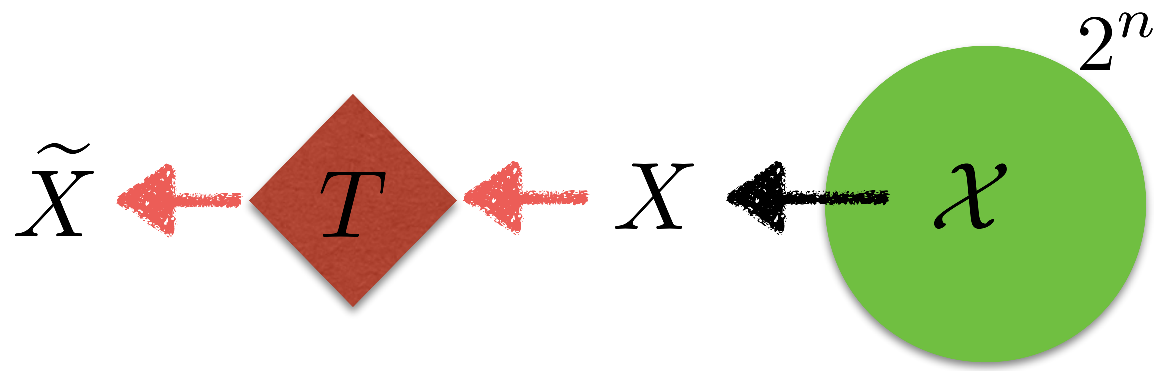

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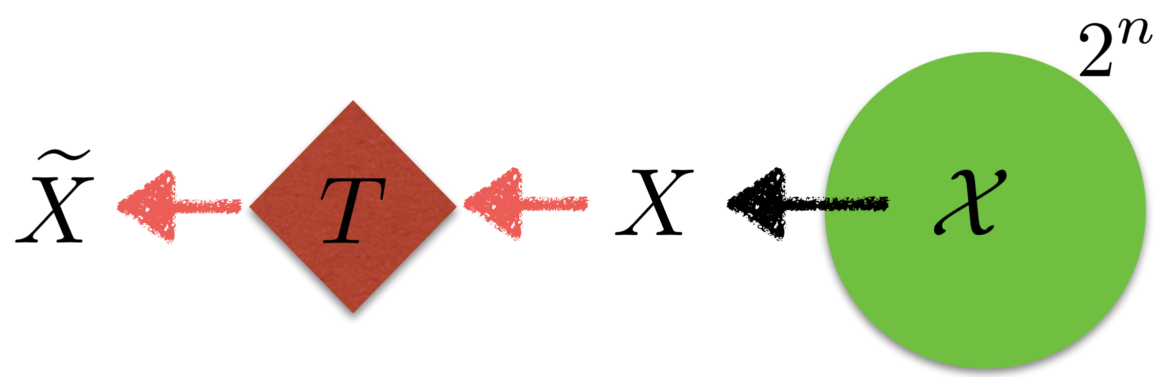

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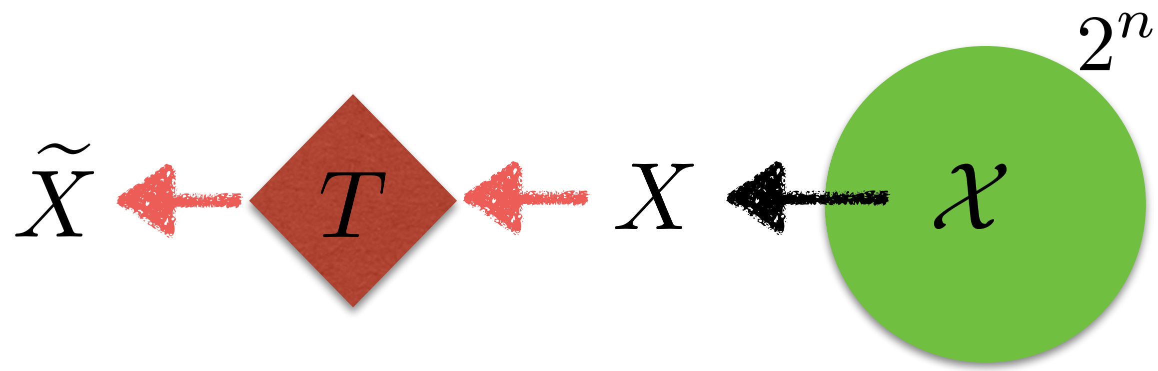

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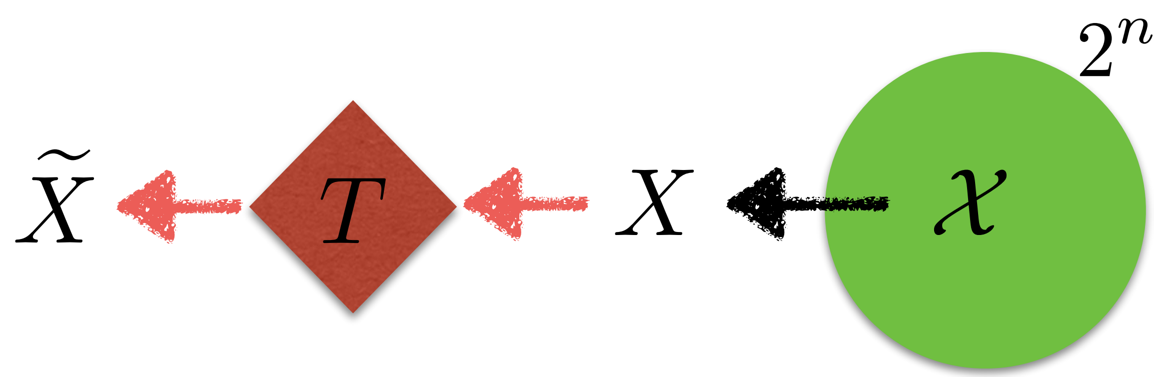

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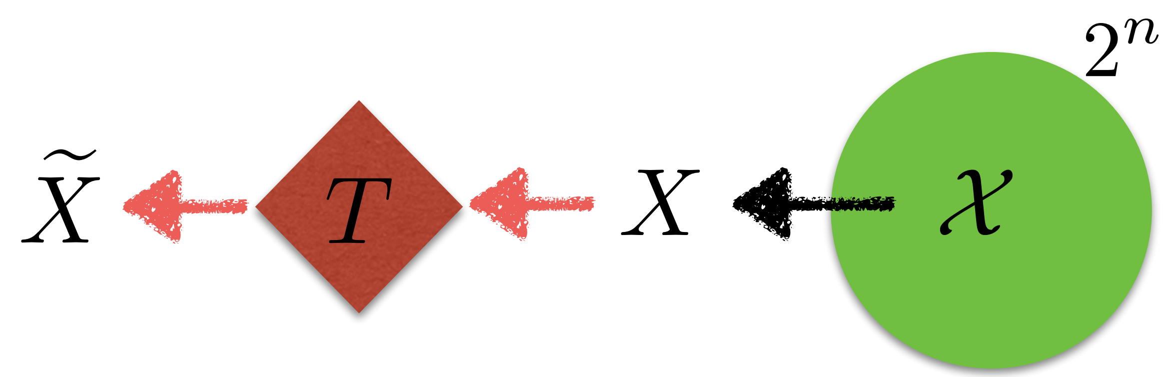

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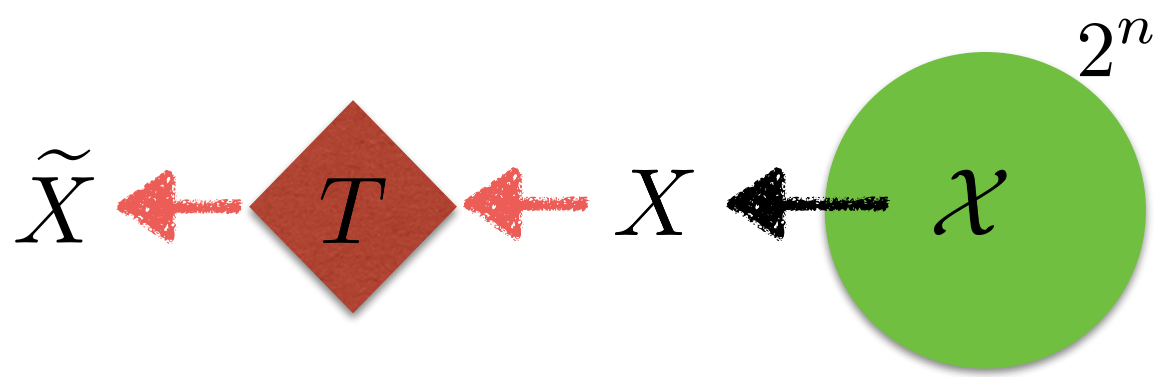

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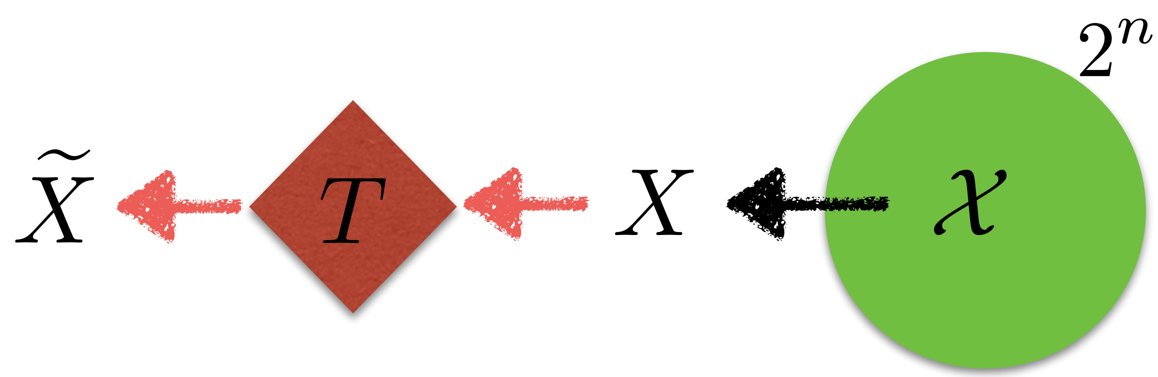

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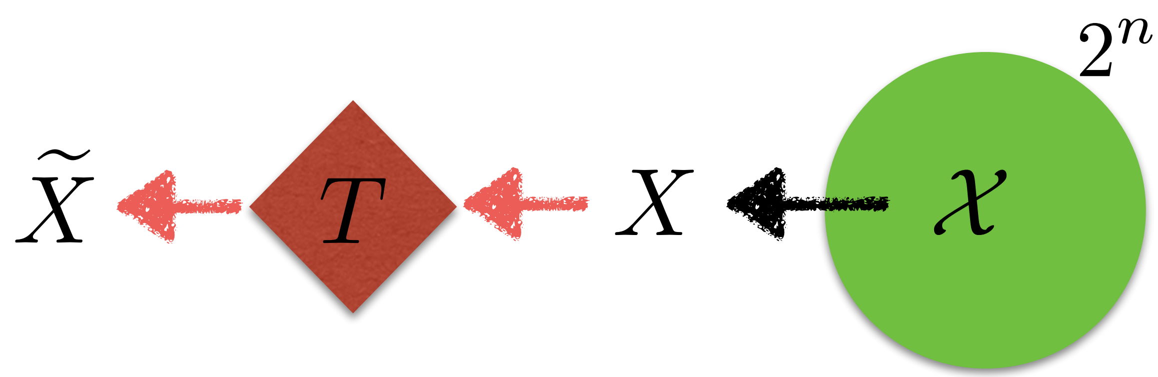

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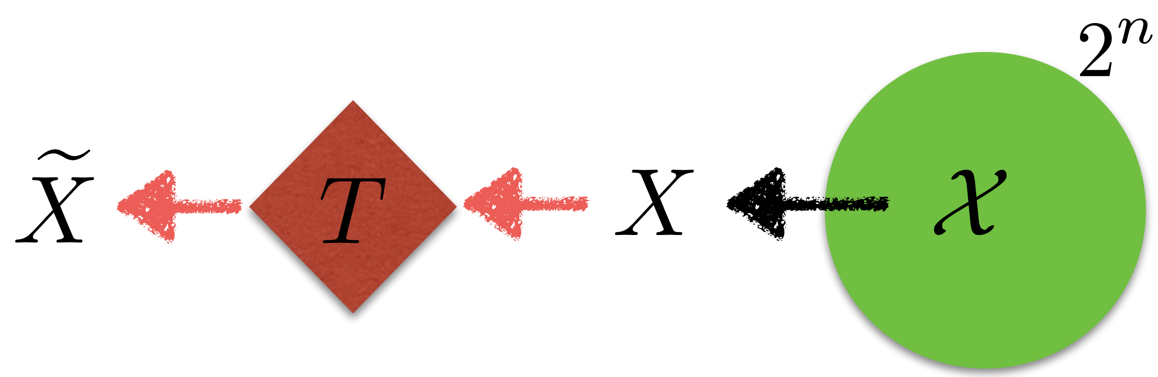

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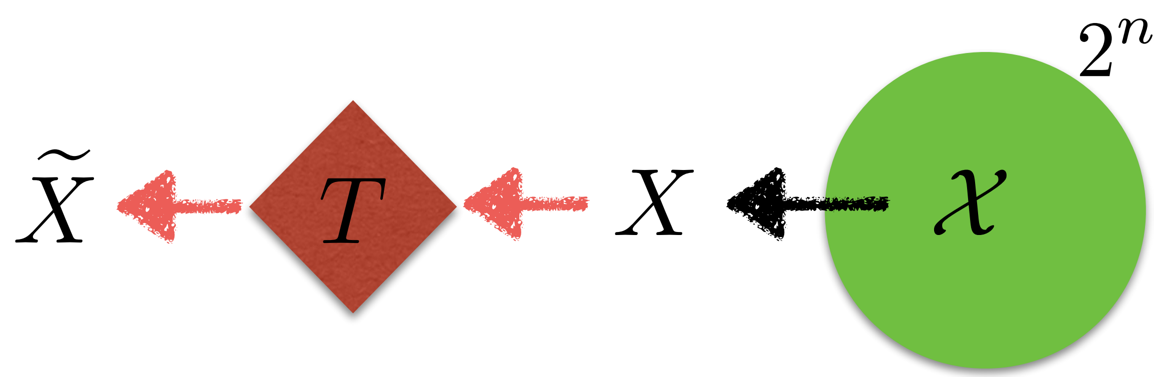

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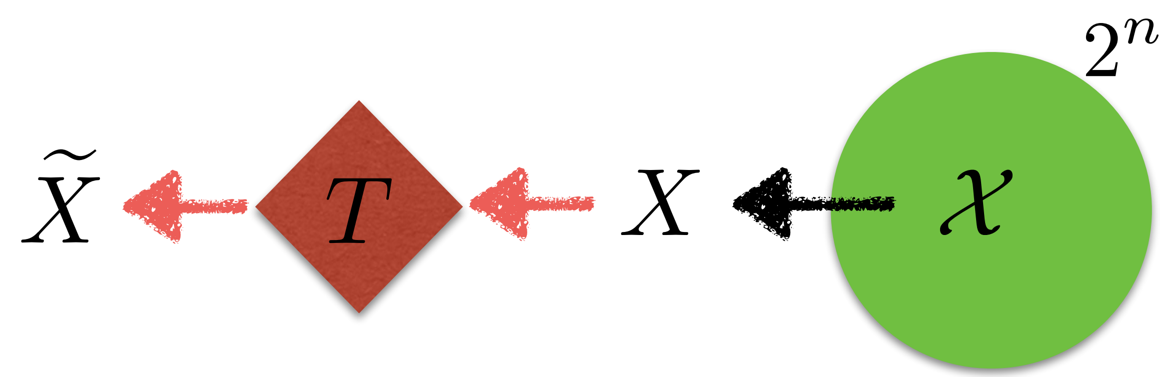

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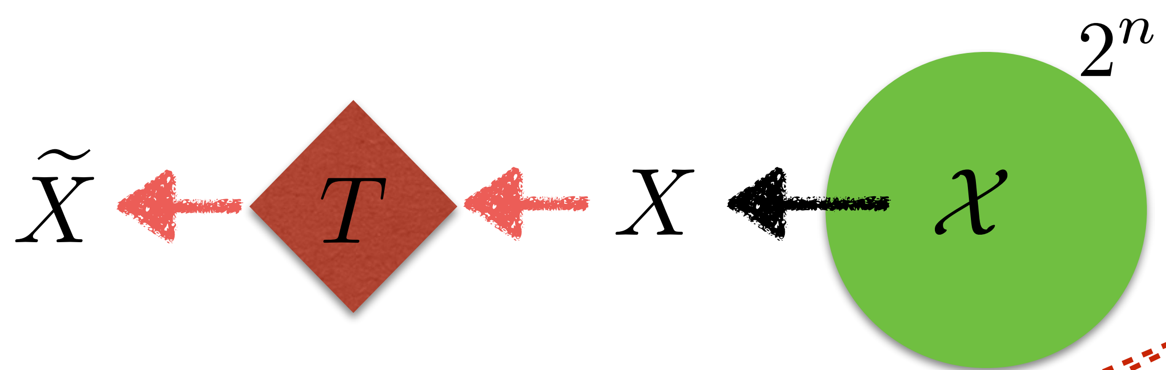
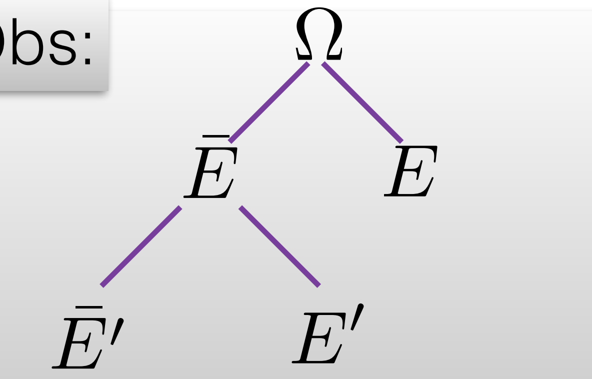


Obs:



$\bar{E} \wedge E' \subseteq E \vee E'$

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Thank You !